
FROM IDEATION TO SUBSTANCE: A METAPHOR-EMBODIED SIMULATION APPROACH TO UNDERSTANDING MATHEMATICAL CONCEPTS

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ABSTRACT

Mathematical concepts are often communicated through symbols, equations, graphs, and formal procedures, which can make abstract relationships difficult to connect with familiar experience. Embodied cognition emphasizes the role of perception, action, gesture, and bodily interaction in cognition, while conceptual metaphor explains how structure from a familiar experiential domain can support understanding of an abstract mathematical domain. This study presents an integrated Metaphor-Embodied Simulation Approach that connects these perspectives. A conceptual literature-synthesis and framework-development methodology was used to examine work on embodied cognition, mathematical cognition, conceptual metaphor, embodied simulation, enactive metaphorising, action and language, gesture, and technology-supported embodied learning. The synthesis produced a framework linking embodied experience, conceptual metaphor, metaphorical mapping, embodied simulation, mathematical representation, and conceptual understanding. It also produced structured algorithms for designing metaphor-embodied mathematical activities and assessing explanation, formal representation, transfer, metaphor-related misconceptions, and independent reasoning. The results indicate that embodied experience can serve as a meaningful starting point for mathematical thought when it is explicitly connected to mathematical language, representation, reflection, and gradual abstraction. The framework is a conceptual synthesis rather than evidence of a single experimentally established cognitive mechanism. It provides a reproducible basis for future empirical testing and for the design and assessment of embodied mathematical learning activities.

KEYWORDS: Embodied cognition; mathematical cognition; conceptual metaphor; embodied simulation; metaphorical mapping; mathematical representation; conceptual understanding.

1. INTRODUCTION

Mathematical understanding is more than recalling formulas or producing correct answers. It involves meaningful relationships among quantities, representations, actions, and mathematical ideas. Research on mathematical cognition therefore considers how people construct and use mathematical knowledge. The reviewed literature shows that mathematical thinking can involve bodily and sensorimotor resources, including perception, movement, gesture, and action (Soylu, 2016; Tran et al., 2017). This makes mathematical understanding important not only as a formal skill but also as a cognitive process grounded in experience. A major challenge is the abstract nature of mathematics. Symbols, equations, graphs, and formal language may represent relationships that have no obvious physical referent. A learner may consequently manipulate a representation without fully understanding its meaning. The literature on conceptual metaphor describes ways in which familiar physical or everyday experiences can provide a basis for understanding mathematical concepts (Kunioka, 2009.; Hotomski, 2013). Embodied approaches offer another way to connect experience with mathematical meaning. Manipulatives, gestures, and whole body actions can provide additional representations of mathematical ideas, although their value depends on how closely the activity is connected to the intended concept (Tran et al., 2017). The present study contributes a conceptual approach that brings conceptual metaphor and embodied simulation together. It proposes a pathway from embodied experience, through metaphorical mapping and simulation, toward formal mathematical representation and conceptual understanding.

1.1 Background and Problem Context

Mathematical cognition concerns how people perceive, represent, reason about, and work with mathematical ideas. The reviewed literature presents mathematics as an important domain for embodied cognition because mathematical thinking can involve relationships between bodily experience, perception, action, and abstract representation. Soylu (2011) describes mathematical cognition as embodied when mathematical processes are grounded in simulations of sensorimotor processes. Tran et al. (2017) similarly describe embodied cognition through interactions between the body and environment and review manipulatives, gestures, and whole body movement in mathematics. This research focuses on how these

ideas can be connected with conceptual metaphor to explain movement from familiar experience toward abstract mathematical understanding.

1.2 The Problem of Abstract Mathematical Concepts

Mathematics is frequently communicated through symbols, equations, diagrams, graphs, and specialized language. These representations are useful, but their meaning may not be immediately clear to every learner. Hotomski (2013) explains that abstract mathematical concepts can become disconnected from the experiences and patterns from which they develop. A learner may therefore manipulate an equation without fully understanding the relationship it represents. Kunioka (2009) describes grounding metaphors as connections between physical or everyday experience and mathematical concepts. The central problem can therefore be viewed as a connection between experience and representation rather than as a problem of symbols alone.

1.3 From Concrete Experience to Abstract Mathematical Thought

The movement from concrete experience to abstract thought is central to embodied mathematics. Tran et al. (2017) explain that physical movements can provide additional cues for representing and retrieving knowledge and can connect concrete referents with abstract concepts. Soylu (2011) uses finger counting to illustrate how an overt action can gradually become an internal motor simulation.

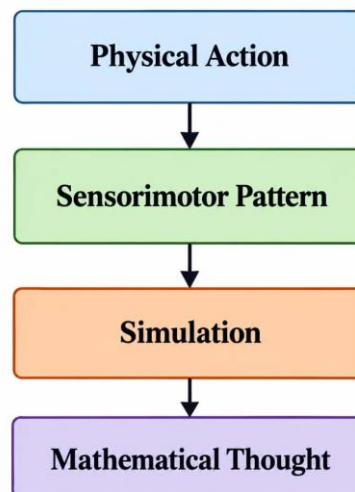


Figure 1: Suggests A Possible Progression.

Khatin-Zadeh et al. (2022) also discuss motion, gesture, and embodied action in relation to numbers, arithmetic, functions, and graphical representations. The important issue is

therefore how bodily experience remains meaningfully connected to mathematical representation as thinking becomes more abstract.

1.4 Conceptual Metaphors in Mathematics

Conceptual metaphor provides a way of explaining how familiar experience can structure understanding of abstract mathematical ideas. A metaphorical relationship involves a source or base domain and a target domain. Knowledge from the source domain is mapped onto the target domain (Soylu, 2011). Kunioka (2009) distinguishes grounding metaphors, which connect everyday experience with mathematical concepts, from linking metaphors, which connect different mathematical concepts.

Khatin-Zadeh et al. (2022) describe mathematical representations involving spatial and motion based structures, including numbers on an axis and movement associated with arithmetic operations. However, metaphor is only a partial mapping, and inappropriate mappings can produce misconceptions (Hotomski, 2013). Metaphor should therefore support, not replace, formal mathematics.

1.5 Embodied Cognition and Mathematical Thinking

Embodied cognition provides the wider theoretical setting for this research. Tran et al. (2017) explain that cognition is influenced by perceptual and physical interactions between the body and the world. Soylu (2011) extends this view to mathematical cognition by discussing simulations of sensorimotor processes. Khatin-Zadeh et al. (2022) likewise argue that motion, gesture, and embodied action can contribute to the grounding of abstract mathematical concepts. Nathan and Walkington (2017) provide a related account through grounded and embodied mathematical cognition (GEMC). Their theory proposes that actions can influence cognitive states and elicit dynamic gestures representing spatial and temporal properties of mathematical entities. They also emphasize that action alone is not sufficient for valid mathematical proof when general properties must be expressed; action needs to be coordinated with language. This supports the view that embodiment should complement, rather than replace, mathematical language and formal representation.

1.6 Need for an Integrated Approach

The reviewed literature connects embodiment, metaphor, action, gesture, and mathematical cognition, but these ideas are often presented through different theoretical routes. Soylu (2011) identifies the need to bridge empirical accounts of bodily processes with conceptual-metaphor accounts and proposes embodied simulation as a possible way to provide a more

connected explanation. Tran et al. (2017) also caution that embodied learning depends on design, guidance, context, and the relationship between activity and learning goals.

1.7 Purpose of the Research

The purpose of this research is to develop a conceptual framework explaining how conceptual metaphor and embodied simulation can be brought together to support understanding of mathematical concepts. The research synthesizes the reviewed literature on embodied cognition, mathematical cognition, conceptual metaphor, gesture, action, and enactive metaphorising. It seeks to clarify how familiar embodied experience can be mapped onto an abstract mathematical idea, how simulation may carry aspects of earlier sensorimotor experience into later thinking, and how these processes can be connected with formal mathematical representation. The resulting framework is intended to guide future research and the design of embodied mathematical experiences.

1.8 Scope of the Research

The research focuses on mathematical understanding from the perspectives of embodied cognition, conceptual metaphor, and embodied simulation. It covers manipulatives, gestures, bodily actions, motion based representations, and selected digital embodied experiences discussed in the literature. Mathematical areas considered include numbers and arithmetic, spatial and geometric ideas, functions and graphical representations, and selected algebraic and calculus ideas where directly supported by the sources. The research is conceptual rather than experimental. It does not report new participant data, statistical tests, or causal effects. Instead, it integrates the supplied literature and proposes a framework connecting embodied experience with mathematical representation. This is consistent with the unresolved questions identified by Tran et al. (2017) concerning the design, implementation, and assessment of embodied mathematical learning.

2. Literature Review

The present study is grounded in a group of related theories rather than in one theory alone. The common point is that mathematical thinking can be connected to bodily experience, action, metaphor, language, and simulation. The theories also provide different explanations of how an experience that begins in the physical world can become part of more abstract mathematical thought.

2.1 Embodied Cognition

Embodied cognition holds that cognition is influenced by the body and by its interaction with the environment. Tran et al. (2017) describe embodied cognition in relation to perception, physical interaction, action, and cognition. Their review shows that mathematical learning can involve manipulatives, gestures, and whole body movement. The learner does not only receive mathematical information; the learner can interact with objects, produce movements, perceive the consequences, and use these experiences while thinking. The theory is useful for the present research because it provides the starting point for connecting physical experience with mathematical meaning. For example, a learner working with objects to represent quantity can see, touch, move, separate, and combine those objects. The physical activity can provide a perceptual and action-based context for a mathematical relationship. In a classroom in North-West Nigeria, the same principle could be applied with locally available objects such as bottle tops, sticks, stones, or counters, provided that the teacher clearly connects the activity to the intended mathematical representation. The theoretical point is not that a particular material automatically teaches a concept, but that bodily interaction can form part of the learning experience.

2.2 Mathematical Cognition

Mathematical cognition concerns the processes involved in understanding and using mathematical ideas. Soylu (2011) describes mathematical cognition as embodied when mathematical processes are grounded in simulations of sensorimotor processes. He identifies empirical work on number processing, arithmetic and related mathematical processes, alongside conceptual metaphor research that explains how physical domain knowledge can be used to understand abstract mathematical ideas.

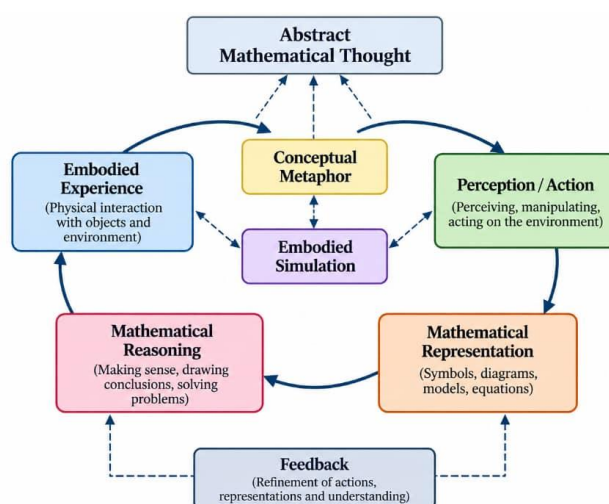


Figure 2.1: Mathematical Cognition.

For example, a learner who moves counters into two groups is not simply performing a physical task. The grouping can provide an experiential basis for quantity and addition. Later, the learner can reason about the relationship using numbers and symbols without physically moving the counters. This illustrates the research interest in how mathematical thought can develop across levels of representation.

2.3 Conceptual Metaphor Theory

Conceptual metaphor theory explains how knowledge from one domain can structure understanding in another domain. In mathematical cognition, Soyly (2011) describes a conceptual metaphor as a link between a physical source domain and an abstract target domain. Kunioka (2009) distinguishes grounding metaphors, which connect everyday experience with mathematical concepts, from linking metaphors, which connect mathematical concepts with one another.

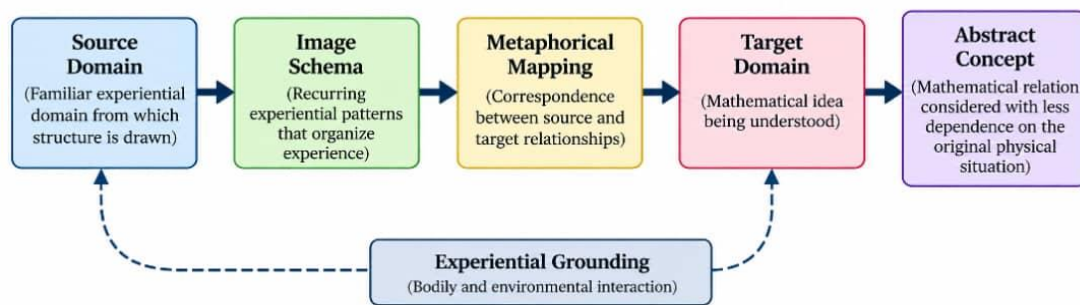


Figure 2.2: Conceptual Metaphor Theory.

The Hotomski thesis also emphasizes that metaphorical mappings are partial and can be misleading if their limitations are not recognized (Hotomski, 2013). Therefore, conceptual metaphor in the present research is treated as a bridge to mathematical understanding, not as a substitute for formal definitions, symbols, procedures, or proof.

2.4 Embodied Simulation

Embodied simulation is particularly important to the present research because it provides a possible mechanism connecting earlier bodily experience with later conceptual processing. Soyly (2011) explains that simulation theories treat cognitive processes as simulations of sensorimotor processes. He illustrates this with finger counting: counting can begin as visible finger movement, become less overt, and eventually operate as a motor simulation without observable movement.

Soylu (2011) further discusses multimodal sensorimotor networks and the idea that experience can establish schematic patterns that later support perceptual, proprioceptive, or introspective simulation. In mathematical cognition, this offers a way to think about how an earlier physical experience can remain relevant when the learner is no longer directly interacting with the original objects.

For example, after repeatedly representing movement along a number line with the body or hand, a learner may later imagine movement from one position to another when solving an arithmetic problem. The physical movement is no longer required, but a related sensorimotor pattern may still provide structure for thought. The present research uses this idea carefully: embodied simulation is proposed as a theoretical link between embodied experience and conceptual processing, not as a claim that every mathematical thought must involve overt movement.

2.5 Grounded and Embodied Mathematical Cognition

Nathan and Walkington (2017) developed grounded and embodied mathematical cognition (GEMC) to explain how action can support mathematical reasoning. Their account emphasizes action-cognition transduction: actions can serve as inputs that influence cognitive states. Their work focuses particularly on geometry, dynamic depictive gestures, mathematical insight, and proof.

An important contribution of GEMC is its treatment of action and language as related but distinct resources. Nathan and Walkington (2017) argue that action can foster mathematical insight, but action alone is not sufficient for valid proof when general properties must be expressed. Language helps learners propositionalize those properties. Their work therefore prevents an overly simple interpretation of embodied learning in which movement is assumed to be sufficient.

For the present approach, this distinction is important. A learner may physically arrange objects or trace a geometric relationship, but the experience must also be connected to mathematical language and representation. Thus, the developed framework treats bodily action, metaphor, simulation, and formal mathematical language as complementary components.

2.6 Enactive Metaphorising

Soto-Andrade (2018) presents enactive metaphorising as an approach in which mathematical metaphors are bodily acted out. His chapter gives examples involving geometry and randomness and argues that the way learners metaphorise and enact a mathematical situation can shape emerging ideas and insights. The approach therefore places bodily enactment

inside the process of mathematical meaning-making rather than treating movement as an additional activity after the mathematical idea has already been formed.

The distinction is useful for this research because it connects conceptual metaphor directly with action. For instance, if a mathematical relation is understood through movement, learners can enact the relevant spatial or relational structure before expressing it formally. A scenario for a classroom could involve learners physically positioning themselves along a marked line to represent ordered numerical positions. The enactment can then be discussed using mathematical language and translated into a number line representation.

Soto-Andrade's material includes learners from different educational backgrounds and uses examples from problem posing and solving. The present research does not claim that these examples establish a universal effect. Instead, it draws from the enactive perspective to explain why bodily enactment is relevant when a metaphor is used to structure mathematical meaning.

2.7 Relationship Between Metaphor and Embodied Simulation

The central theoretical contribution of the present research is to connect conceptual metaphor with embodied simulation. The two ideas address different parts of the same problem. Conceptual metaphor explains how structure from a familiar experiential domain can be mapped onto an abstract mathematical domain. Embodied simulation provides a possible account of how aspects of sensorimotor experience can remain involved in later conceptual processing (Soylu, 2016).

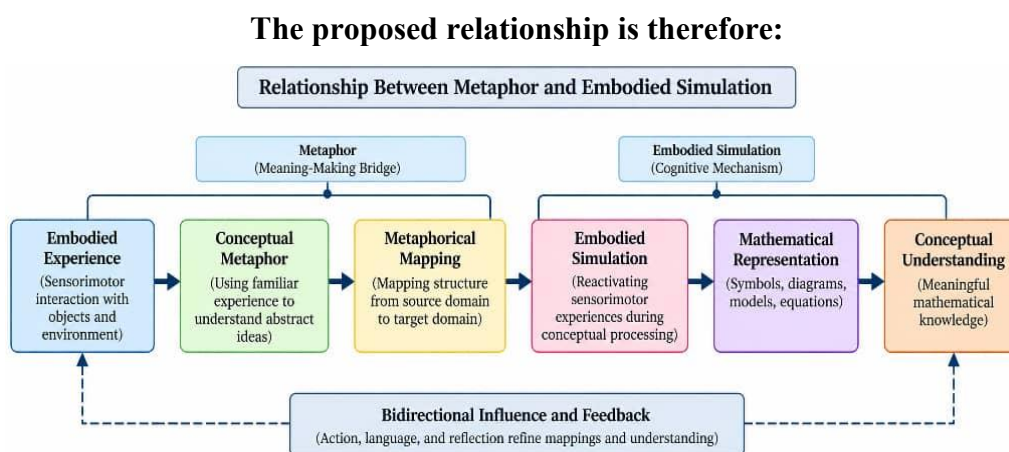


Figure 2.3: Relationship Between Metaphor And Embodied Simulation.

This relationship integrates the literature without claiming that the sequence has already been empirically established as a single mechanism. It provides a conceptual model for future

testing and instructional design. It also accommodates the findings of Nathan and Walkington (2017), because action and language remain connected, and Soto-Andrade (2018), because mathematical metaphor can be enacted bodily. The model therefore treats metaphor as a meaning-making bridge and simulation as a possible cognitive mechanism through which embodied experience can continue to support mathematical thought.

2.8 Review of Embodied Approaches to Mathematical Thinking

Following the broad organization of Tran et al. (2017), this section reviews major forms of embodied mathematical experience. The review moves from direct interaction with objects to gestures, whole body movement, action, language, and technology-supported environments. The categories should not be treated as completely separate because one learning activity can involve several forms of embodiment at the same time.

2.8.1 Concrete Manipulatives

Concrete manipulatives allow learners to physically interact with objects that stand for mathematical quantities or relationships. Tran et al. (2017) identify manipulatives as an important area in the embodied mathematics literature. Their value lies in providing perceptual and physical access to ideas that may otherwise be represented only symbolically. For example, ten bottle tops can be arranged into two groups of five. The learner can combine the groups and then connect the physical result with $5 + 5 = 10$. In a North-West Nigerian classroom, inexpensive locally available objects could support such an activity where appropriate. The important design principle is that the physical object must be explicitly connected to the mathematical relationship. A manipulative should not be treated as effective simply because it is concrete.

2.8.2 Hand and Finger Gestures

Hand and finger gestures provide a more flexible form of embodiment. They can support counting, pointing, tracing, grouping, and spatial representation. Finger counting is a particularly direct example: Soylu (2011) uses it to illustrate how an overt motor activity can gradually become a motor simulation. Pointing can direct attention to a particular number, point, or part of a graph. Tracing can reproduce the path or shape of a mathematical representation. Grouping gestures can represent the formation of sets, while spatial gestures can show direction, distance, rotation, or change.

A classroom scenario could involve a teacher drawing a number line and asking learners to move a finger from -2 to -7 while explaining the arithmetic operation. Khatin-Zadeh et al. (2022) specifically discuss understanding arithmetic operations as movement along an axis. Symbolic gestures can also accompany mathematical language, allowing an abstract relation

to be represented through a bodily action. These examples illustrate that gesture can function as part of mathematical representation rather than merely as classroom communication.

2.8.3 Whole Body Movement

Whole body movement extends embodiment beyond the hands. Tran et al. (2017) review whole body movement as another form of embodied mathematical activity. Such activities can make spatial, directional, relational, or dynamic properties physically available to learners.

For example, learners can stand at different positions on a large number line drawn on the classroom floor. Moving forward or backward can represent changes in numerical position. In geometry, learners could use their bodies to form lines, angles, or spatial relationships before representing those relationships on paper. In North-West Nigeria, such activities may be adapted to available classroom or school spaces without requiring specialized equipment. However, the mathematical purpose of the movement should remain explicit. Tran et al. (2017) caution that movement itself does not guarantee learning; design and instructional guidance remain important.

2.8.4 Action-Based Mathematical Learning

Action based learning emphasizes purposeful physical interaction as part of mathematical reasoning. Nathan and Walkington (2017) describe actions as inputs that can influence cognition through action-cognition transduction. Their work shows how directed actions and dynamic gestures can contribute to mathematical insight, particularly in geometry.

An example is a learner physically manipulating a shape to investigate a geometric relationship and then explaining the observed relationship mathematically. The action provides experience, while the subsequent reasoning identifies the mathematical property. This distinction is important because action should not be confused with learning. The action must be linked to the mathematical question and followed by interpretation, representation, or explanation.

2.8.5 Language and Embodied Action

Language provides an important connection between embodied activity and formal mathematical reasoning. Nathan and Walkington (2017) argue that action can support insight but that valid proof requires coordination with language when general mathematical properties are expressed. Thus, a bodily experience can initiate or support an idea, while mathematical language can make the relationship explicit.

For example, after learners move along a number line, the teacher can ask them to describe what changed, identify the starting and ending values, and write the corresponding equation.

The sequence becomes:

Action → observation → mathematical language → symbolic representation.

This approach is consistent with the wider literature because embodied learning is not presented as a replacement for formal mathematics. Instead, bodily experience can provide a meaningful context in which mathematical language and symbols acquire clearer connections.

2.8.6 Digital Embodied Experiences

The Tran et al. (2017) review extends the discussion of embodied cognition to technology supported experiences. Tran et al. (2017) discuss digital touch devices, motion sensors, location sensors, three dimensional technologies, and virtual reality as technologies that can support direct manipulation and immersive experiences. Digital environments can therefore provide forms of embodiment that are difficult to reproduce with conventional materials. A touch interface can allow a learner to drag a point on a graph and observe how the representation changes. Motion sensing can connect body movement with mathematical events. An interactive environment can allow learners to manipulate shapes or quantities, while a virtual environment can provide spatial experiences that would be difficult to construct physically. These examples should be understood as affordances rather than automatic improvements. The Tran et al. (2017) review emphasizes that the design of the activity, the mathematical goal, learner guidance, and assessment remain important. For the present research, digital embodiment is included as an extension of the same theoretical pathway: an embodied interaction can provide experience, metaphor can organize its mathematical meaning, and embodied simulation may help connect the experience with later conceptual processing.

3. METHODOLOGY

3.1 Research Design

The study used a conceptual literature-synthesis and framework-development design. This design was selected because the purpose was to integrate established ideas from embodied cognition, mathematical cognition, conceptual metaphor, embodied simulation, action, gesture, language, and digital embodiment into a coherent framework for mathematical learning. The study did not collect new participant, classroom, experimental, or statistical

data; instead, the analytical output was a theoretically grounded framework and a set of reproducible design and assessment procedures.

3.2 Literature Base and Source Selection

The core literature base comprised the Tran et al. (2017) review and the supporting works already used throughout the manuscript, including Soylu (2011), Kunioka (2009), Nathan and Walkington (2017), Soto-Andrade (2018), Hotomski (2013), and Khatin-Zadeh et al. (2022). Additional peer-reviewed sources were incorporated to strengthen specific claims about grounded cognition, gesture, mathematical learning, and transfer, including Barsalou (2008), Glenberg (2010), Alibali and Nathan (2012), Novack et al. (2014), Cook et al. (2017), Hostetter and Alibali (2008), and Seccia and Goldin-Meadow (2024). Sources were retained when they directly addressed the relationship among bodily experience, action, gesture, simulation, mathematical representation, language, or learning.

3.3 Analytical Procedure

The literature was organized into recurring themes: (i) embodied interaction and grounding, (ii) conceptual metaphor and source-target mapping, (iii) embodied simulation, (iv) gesture and action, (v) mathematical language and formal representation, (vi) digital embodied experience, and (vii) assessment and transfer. The study then compared the roles assigned to these themes across the reviewed sources and identified complementary rather than identical functions. Conceptual metaphor was treated as a meaning-making bridge, while embodied simulation was treated as a possible mechanism through which aspects of prior sensorimotor experience may remain relevant during later conceptual processing.

3.4 Framework Construction

The framework was constructed by arranging the recurring relationships into a coherent instructional pathway: embodied experience → conceptual metaphor → metaphorical mapping → embodied simulation → mathematical representation → conceptual understanding. The pathway was then expanded into two algorithms. The first specifies how an instructor can design a metaphor embodied mathematical activity, including selection of the mathematical concept, source experience, mapping, activity, language, formal representation, abstraction, and assessment. The second specifies an assessment cycle based on observation, explanation, formal representation, transfer, misconception identification, feedback, and reassessment.

3.5 Validity and Interpretive Limits

The synthesis was checked against the scope and cautions of the reviewed literature. In particular, the analysis did not treat physical movement as automatically producing learning,

did not assume that every mathematical concept has a suitable bodily metaphor, and did not treat metaphorical mappings as complete equivalences. These limits are important because the literature identifies design, guidance, language, assessment, and metaphorical boundaries as conditions affecting the usefulness of embodied approaches.

The literature synthesis produced four connected outputs: an integrated metaphor-embodied simulation framework, a structured activity-design algorithm, an assessment algorithm, and a set of instructional principles linking action, gesture, language, formal representation, abstraction, and transfer. The results are presented as conceptual and design outcomes of the synthesis rather than as statistical estimates of learning effects.

4. RESULTS: Metaphor-Embodied Simulation Framework

The metaphor-embodied simulation approach is intended to explain how learners can move from familiar bodily experience to more formal mathematical understanding. It does not treat physical activity as an end in itself. Instead, the approach organizes experience, metaphorical mapping, simulation, language, and mathematical representation into a connected learning process. This is consistent with the literature showing that embodied cognition can involve perception, action, gesture, and interaction with the environment, while conceptual metaphor can connect familiar experience with abstract mathematical ideas (Soylu, 2016; Tran et al., 2017).

The approach begins with an embodied situation that has a clear mathematical purpose. The learner first experiences a quantity, movement, spatial relation, transformation, or other mathematical structure through objects or bodily action. The teacher then helps the learner identify the mathematical relationship represented by the experience. At this stage, conceptual metaphor provides a bridge between the familiar source domain and the mathematical target domain. Embodied simulation is then considered as a possible way through which aspects of the earlier sensorimotor experience can remain available during later mathematical thinking (Soylu, 2011).

For example, consider addition using bottle tops. A teacher gives a learner five bottle tops and another five and asks the learner to place them in two groups. The learner combines the groups and counts ten objects. The physical experience represents collection and combination. The teacher then connects the experience to the expression $5 + 5 = 10$. With repeated use, the learner can later reason about the same relationship without physically moving the bottle tops. The original action therefore becomes a possible experiential basis for a more abstract representation.

A second scenario can involve integers. Learners stand on a number line drawn on the classroom floor. A learner starts at -2 and moves seven positions to the left, arriving at -9. The movement provides an embodied representation of direction and change. The teacher then asks the learner to state the corresponding operation and write $-2 - 7 = -9$. Later, the learner may imagine movement along the number line without physically walking. This scenario illustrates the proposed connection between action, metaphor, simulation, language, and symbolic reasoning. Motion-based interpretations of arithmetic and mathematical concepts are also discussed in the reviewed literature (Khatin-Zadeh et al., 2022).

The same principle can be applied to geometry. Learners may use their arms to form an angle, compare angles, and then represent the relationship with diagrams and mathematical terms. Nathan and Walkington (2017) emphasize that action must be coordinated with language when general mathematical properties or proof are expressed.

The approach can also be adapted to classrooms in North-West Nigeria using low cost materials. Bottle tops, sticks, stones, ropes, paper shapes, and chalk markings can provide concrete or spatial representations when appropriate. The central requirement is not the sophistication of the material but the quality of the mapping between the experience and the intended mathematical idea. This follows the caution in Tran et al. (2017) that embodied activity requires appropriate design and guidance.

4.1 Framework for Designing Metaphor Embodied Mathematical Activities

The instructional design translates the framework into a sequence for developing mathematical learning activities. It contains seven stages: identify the concept, select an embodied source, construct the metaphorical mapping, enact the experience, connect it to language and representation, support abstraction and simulation, and assess understanding.

Stage 1: Identify the mathematical concept. The teacher begins with a clear target, such as addition, integer operations, angle relationships, functions, or graphical change.

Stage 2: Select an embodied source experience. The teacher identifies an everyday, physical, spatial, or action based experience with relevant structure. Objects can represent addition, movement can represent integers, and bodily positioning can represent geometry.

Stage 3: Construct the metaphorical mapping. The teacher identifies which features of the source correspond to the mathematical target. For example, movement to the right can represent increase on a number line. The limits of the metaphor should also be identified because a partial mapping can mislead learners (Hotomski, 2013).

Stage 4: Enact or manipulate. Learners perform the action, manipulate objects, gesture, or interact with a representation. The action should be purposeful and linked to the mathematical

question. Tran et al. (2017) emphasize that movement or technology does not automatically produce learning.

Stage 5: Connect action to language and representation. Learners describe what they did, explain the relationship, construct a representation, and write the mathematical expression. This links embodied experience with formal mathematical communication (Nathan & Walkington, 2017).

Stage 6: Support abstraction and simulation. The teacher gradually reduces dependence on physical material. Learners solve related problems using diagrams, symbols, mental imagery, or imagined movement. This reflects Soylu's (2011) discussion of the transition from overt finger movement toward less observable motor simulation.

Stage 7: Assess understanding. Assessment should determine whether learners can explain the relationship, use formal representations, transfer the idea, and recognize the limits of the metaphor. Physical participation alone should not be treated as evidence of learning (Tran et al., 2017).

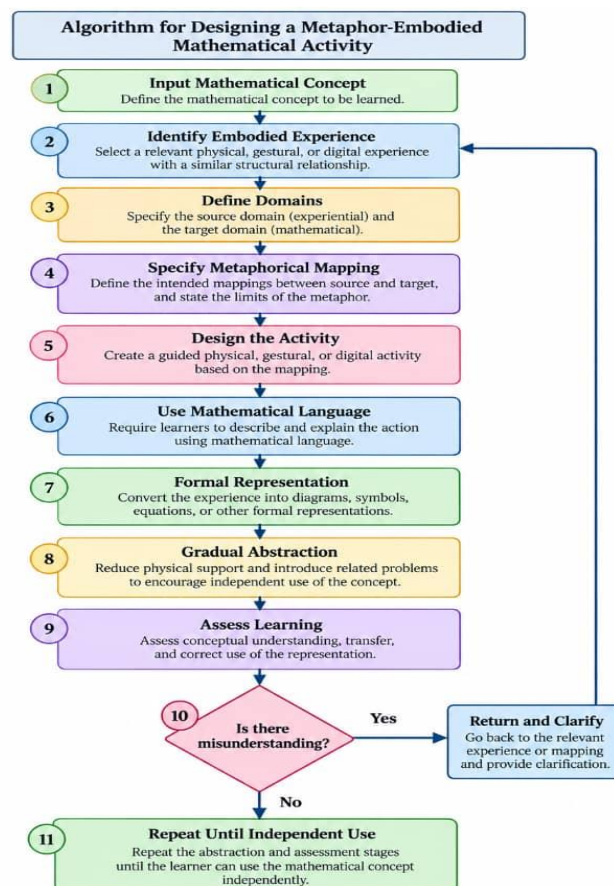


Figure 4.1: Algorithm For Designing a Metaphor Embodied Mathematical Activity.

4.2 Role of Gesture, Action, and Language

Gesture, action, and language have complementary roles in the developed approach. Gesture provides a flexible way to represent mathematical relations through the hands, fingers, or body. Action provides direct interaction with objects, space, or mathematical representations. Language allows learners to identify, explain, generalize, and formalize the relationships experienced through action. The gesture-as-simulated-action literature also provides a direct theoretical connection between visible gesture, perceptual-motor simulation, and mental imagery (Hostetter & Alibali, 2008).

Gesture can be particularly useful when a mathematical idea has spatial or directional structure. A learner can point to a number, trace a graph, show the direction of movement, indicate an angle, or demonstrate grouping. Finger counting also illustrates how an overt bodily activity can become less visible while remaining relevant to mathematical processing (Soylu, 2011). In this sense, gesture can function both as an external representation and as part of a possible route toward internal simulation. Research on mathematical gestures also indicates that gestures can externalize spatial structure and support mathematical thinking and learning (Alibali & Nathan, 2012; Novack et al., 2014).

Action provides the experiential component of the framework. Learners can move objects, change positions, manipulate shapes, or enact relationships. Nathan and Walkington (2017) describe action-cognition transduction as a process in which action can serve as an input to cognition. Their work also shows why action should be connected to interpretation. An action that is not linked to a mathematical question may remain simply a physical activity.

Language performs a different but connected function. After an action, learners need opportunities to describe what happened, identify the mathematical relationship, and express it using appropriate terminology and symbols. For example, after moving from -2 to -9 on a floor number line, a learner can state: "I moved seven places to the left, so I subtracted seven." The teacher can then connect the statement to the symbolic expression $-2 - 7 = -9$.

This sequence is especially important for advanced mathematics. Nathan and Walkington (2017) argue that action may support insight, but valid proof requires mathematical language to express general properties. Therefore, the developed approach does not replace formal reasoning with movement. Instead, it uses action and gesture as resources that can support the development and interpretation of mathematical representations.

The three components can also operate together. In a geometry lesson, for example, learners may use their arms to form two angles, gesture to compare their size, explain the relationship verbally, and finally represent the relationship using a diagram and mathematical notation. In

a North-West Nigerian classroom, similar activities can be organized with locally available materials and ordinary classroom space. Digital tools may also be used where available, including touch interfaces or motion-sensitive environments discussed by Tran et al. (2017). However, technology should remain an instructional means rather than the learning objective. The developed model therefore gives gesture, action, and language distinct but connected functions: action provides experience, gesture externalizes or highlights structure, language makes relationships explicit, and formal representation supports abstraction and mathematical communication. Together, they provide a practical pathway for testing the broader metaphor-embodied simulation framework.

4.3 Conceptual Understanding and Assessment

The metaphor embodied simulation approach treats conceptual understanding as more than successful participation in a physical activity or correct completion of a single exercise. Understanding should be examined through the learner's ability to connect the embodied experience with mathematical language, representations, reasoning, and transfer. This position follows the caution in Tran et al. (2017) that movement itself is not evidence that learning has occurred. Assessment should therefore examine what the learner can understand and do after the embodied activity.

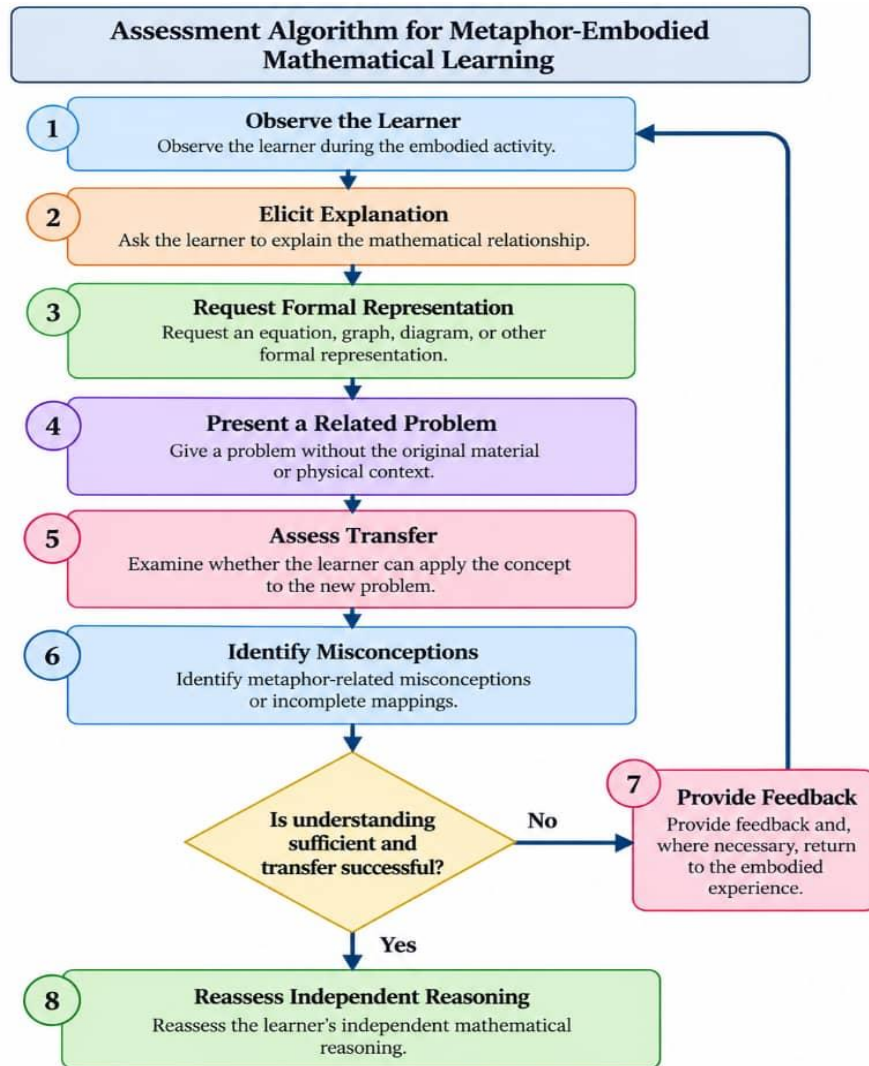
Four related dimensions can guide assessment. First, the learner should be able to explain the mathematical relationship represented by the action or object. Second, the learner should be able to translate the experience into a diagram, equation, graph, or other mathematical representation. Third, the learner should be able to apply the idea to a related problem without depending completely on the original physical activity. Fourth, the learner should recognize important limits of the metaphor rather than treating the source experience as identical to the mathematical concept. The last point is important because metaphorical mappings are partial and can contribute to misconceptions when their limits are not made clear (Hotomski, 2013).

For example, after using bottle tops to represent $5 + 5 = 10$, a teacher should not assess only whether the learner can arrange ten bottle tops. The teacher can instead ask the learner to explain why the two groups represent addition, write the equation, solve a related problem such as $6 + 5$, and explain the relationship without the objects. A learner who can complete these tasks demonstrates a stronger connection between the embodied experience and formal mathematics.

A second scenario involves integers. After learners physically move from -2 to -9 on a floor number line, assessment can include four tasks: describe the movement, identify the

arithmetic operation, represent it symbolically as $-2 - 7 = -9$, and solve a similar problem mentally. The assessment can then ask the learner to explain why movement to the left represents subtraction in the chosen representation. This sequence tests representation, language, reasoning, and transfer rather than physical participation alone.

A useful assessment algorithm is:



This procedure reflects the distinction between embodied activity and mathematical learning. Nathan and Walkington (2017) also show the importance of language when mathematical insight must be expressed as general properties or proof. Consequently, assessment should include verbal or written explanation where appropriate.

4.4 Summary of Framework Outputs

The completed framework has three principal components. First, the theoretical model connects embodied experience with conceptual metaphor, metaphorical mapping, embodied

simulation, mathematical representation, and conceptual understanding. Second, the activity-design algorithm operationalizes this pathway as a repeatable sequence from concept selection through embodied experience, language, formal representation, abstraction, and assessment. Third, the assessment algorithm establishes a feedback cycle in which learner explanation, formal representation, transfer, misconception detection, and independent reasoning are examined before the activity is revised or extended. Together, these outputs convert the theoretical synthesis into a practical and reproducible framework for instructional design and future empirical evaluation.

5. DISCUSSION

The framework brings together ideas that appear across the reviewed literature but are often discussed through separate theoretical perspectives. Embodied cognition emphasizes interaction among body, environment, perception, and action (Tran et al., 2017). Conceptual metaphor explains how structure from familiar experience can be mapped onto an abstract mathematical target (Soylu, 2016; Kunioka, n.d.). Embodied simulation provides a possible account of how sensorimotor experience may remain relevant during later conceptual processing (Soylu, 2011). GEMC further emphasizes the relationship between action, cognition, and language (Nathan & Walkington, 2017).

The main contribution of the developed framework is therefore integrative rather than a claim of a newly established causal mechanism. It proposes that embodied experience can provide a source domain, conceptual metaphor can organize the mapping to a mathematical target, and embodied simulation can provide a possible bridge to later conceptual processing. Mathematical language and formal representation then provide the means for making the relationship explicit and usable in more abstract reasoning.

This framework also explains why the same physical activity can produce different learning outcomes. Consider two learners using a floor number line. One learner simply follows instructions to walk seven steps. Another learner is asked to identify the starting number, explain the direction, connect the movement to subtraction, write the corresponding equation, and later solve the problem without walking. The physical movement is similar, but the mathematical interpretation is different. The developed approach therefore places instructional guidance and reflection at the centre of embodied learning.

The framework also supports the use of low cost materials. In North-West Nigerian classrooms, teachers may not always have access to specialized digital or laboratory equipment. However, the developed approach does not require advanced technology. Locally

available objects, chalk markings, paper shapes, ropes, or classroom space can support suitable mathematical representations. The important issue is whether the activity has a clear mathematical purpose and whether learners are guided from action to representation and abstraction.

The framework should nevertheless be interpreted cautiously. The reviewed sources do not establish that every mathematical concept is best learned through bodily activity, nor do they establish that more embodiment necessarily produces better learning. Tran et al. (2017) specifically identify design, implementation, and assessment as important issues. Hotomski (2013) also indicates that conceptual metaphors are limited and can sometimes mislead learners. The developed approach therefore treats embodiment as a structured learning resource rather than a universal replacement for conventional mathematics instruction.

The model provides a basis for future empirical studies. Researchers could compare different embodied designs, examine how learners move from overt action to less observable simulation, investigate the role of language, and test whether carefully designed metaphorical mappings support transfer to formal mathematical tasks. Such studies would be needed before stronger claims about effectiveness can be made.

6. Implementation Challenges

Although the approach can be implemented with simple materials, several challenges need to be considered. The first is instructional alignment. A physical activity may be enjoyable or visually attractive without representing the intended mathematical structure. Teachers therefore need to begin with the mathematical concept and then select an embodied experience that supports it.

The second challenge is metaphorical limitation. A source experience may correctly represent some aspects of a mathematical idea but not others. For example, a number line can represent numerical order and movement, but physical movement should not be assumed to capture every property of numbers. Teachers should explicitly discuss what the metaphor represents and where it stops.

The third challenge is classroom management and space. Whole-body activities may require additional classroom space, preparation, and time. In some schools, crowded classrooms can make movement difficult. The approach should therefore include scalable alternatives, such as finger gestures, desk-based objects, paper representations, or small-group activities.

The fourth challenge is teacher preparation. Teachers need to understand both the mathematical target and the intended mapping. They also need to know when to reduce

physical support and move learners toward formal representation. Without this transition, learners may remain dependent on the physical material.

The fifth challenge concerns resources and digital access. Tran et al. (2017) describe touch devices, motion sensors, three-dimensional technologies, and virtual environments as possible embodied learning technologies, but such resources may not be available in every setting. The developed framework therefore distinguishes the theoretical principle from the technology used to implement it. Digital tools can extend an embodied experience, but they are not required.

A sixth challenge is assessment. It can be difficult to determine whether an observed gesture or movement reflects mathematical understanding. Assessment should therefore include explanation, representation, transfer, and independent reasoning. This is particularly important because physical participation can occur without deep conceptual understanding.

If any stage is weak, the teacher can revise the activity before extending it. This iterative approach is consistent with the need for careful design and assessment identified in the anchor review (Tran et al., 2017).

7. Future Research Directions

The developed framework is conceptual and requires empirical testing. Future studies should examine whether the pathway from embodied experience, through conceptual metaphor and embodied simulation, to formal mathematical reasoning can be observed in actual learning. This is consistent with Tran et al. (2017), who identify design, implementation, and assessment as important areas for future embodied mathematics research.

One direction is to compare different embodied activities while keeping the mathematical concept constant. For example, a study could compare bottle top manipulation, finger gestures, and digital touch interaction for teaching addition. Another study could examine whether repeated movement along a number line supports later mental reasoning about integers. Soylu (2011) provides a theoretical basis for examining how overt motor activity may become less observable during later mathematical processing.

Future research should also examine the quality of metaphorical mappings. A learner may understand “numbers as points on a line” but incorrectly extend the metaphor to properties it does not represent. Studies should therefore examine both conceptual gains and misconceptions. Kunioka (2009) and Hotomski (2013) provide useful foundations for investigating grounding metaphors, linking metaphors, and their instructional limitations.

A further direction concerns language. Nathan and Walkington (2017) indicate that action can support mathematical insight, while language is important for expressing general mathematical properties and proof. A future study could therefore compare embodied activities with and without structured mathematical explanation.

Research in North-West Nigeria could also examine whether low cost materials such as bottle tops, sticks, ropes, paper shapes, and chalk markings can support carefully designed embodied mathematics activities. Such studies should evaluate learning outcomes rather than assume that the materials themselves produce improvement.

8. CONCLUSION

This paper presents a metaphor-embodied simulation approach for understanding how mathematical concepts may develop from bodily and experiential activity toward formal mathematical reasoning. The framework connects conceptual metaphor, embodied cognition, embodied simulation, action, gesture, language, and mathematical representation.

The central proposition is that embodied experience can provide a meaningful source domain, conceptual metaphor can organize the mapping to a mathematical target, and embodied simulation may help maintain aspects of sensorimotor experience during later thinking (Soylu, 2011). Action and gesture can make mathematical relationships visible, while language and formal representation help learners express, generalize, and abstract those relationships (Nathan & Walkington, 2017).

The framework does not claim that movement or metaphor automatically produces mathematical learning. Rather, it emphasizes deliberate design, clear mapping, guided reflection, formal representation, and assessment. This position is consistent with the broader review by Tran et al. (2017). The developed model therefore provides a basis for future empirical studies and instructional design rather than a completed causal theory.

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